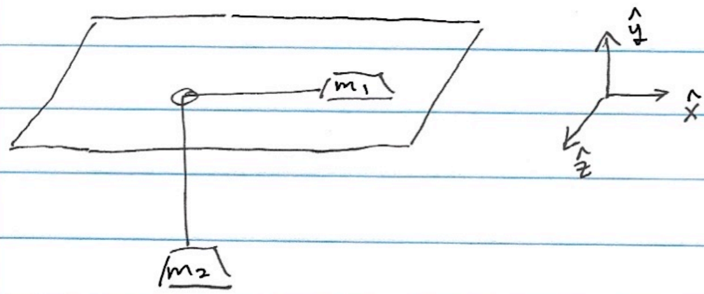
Goldstein 1.21

Evidently the problem is reduced to x-y plane:



m_1 at (x_1, y_1)

m_2 at (x_2, y_2)

By placing the origin at the hole, we have $y_1 = 0$, $x_2 = 0$ always, so the system is described by at most 2 variables x_1, y_2. Yet these two varriable can not serve as our generalized coordinates for they are not independent and are related by

$$dx_1 = dy_2$$

$$\Rightarrow \quad x_1 = y_2 + c, \quad \dot{x}_1 = \dot{y}_2$$

So evidently there is only one generalized coordinate, we will choose it to be x_1. It gives the Lagrangian.

$$L = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{y}_2^2 - m_2 g y_2 = \frac{1}{2}(m_1 + m_2)\dot{x}_1^2 - m_2 g (x_1 - c)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_1} = (m_1 + m_2)\ddot{x}_1, \quad \frac{\partial L}{\partial x_1} = -m_2 g$$

The eqm is $(m_1 + m_2)\ddot{x}_1 + m_2 g = 0$, $\ddot{x}_1 = -\frac{m_2}{(m_1 + m_2)} g$

solving gives $\boxed{x_1 = -\frac{1}{2}\frac{m_2}{(m_1 + m_2)} g t^2}$

The physical significance is that in the absence of friction, the two masses accelerate down / to the hole at acceleration $\frac{m_2}{m_1 + m_2} g$.

If we wish to consider friction, we may introduce dissipative force through some force of Q_j that is generalized force.

Dawdsu Chen

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